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SENIOR SCHOOL

AS/A Level Mathematics

AS/A level Mathematics course structure

All of the content in the AS/A level Mathematics qualification is compulsory and is the same for all examination boards.

Pure Mathematics
66%

Statistics
17%

Mechanics
17%

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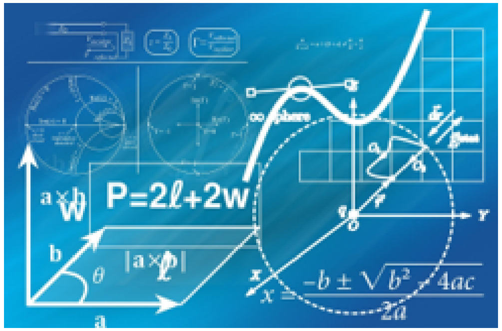
What is Pure Maths?

Methods and techniques which underpin the study of all other areas of mathematics, such as, proof, algebra, trigonometry, calculus, and vectors



The points A and B have coordinates (4,-2) and (10,6) respectively.

Find the equation of the circle that has AB as a diameter.

What is Statistics?

Reaching conclusions from data and calculating the likelihood of an event occurring.

What is the probability of two '100 year floods' happening within the space of 5 years?

What assumptions have you made?

"The majority of private sector organisations believe the use of data analytics will be the most important factor in increasing growth in UK businesses"

Professor Sir Adrian Smith

What is Mechanics?

The modelling of the world around us, the motion of objects and the forces acting on them.

A golfer drives their ball from a tee on horizontal ground so that it has an initial velocity of $50ms^{-1}$ at an angle of 40 degrees above the horizontal.

How far down the fairway will the ball land?

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FPSS entry requirements

AS/A level Mathematics

- At least **grade 6** in GCSE Maths.

This week's lessons

- Lesson 1 – Binomial Expansion (Pure)
- Lesson 2 – Binomial Distribution (Statistics)

BINOMIAL EXPANSION

Lesson 1

Starter Activity

a) Expand $(a+b)^0$

b) Expand $(a+b)^1$

c) Expand $(a+b)^2$

d) Expand $(a+b)^3$

e) Expand $(a+b)^4$

?
?
?
?
?

What do you notice about:

The coefficients: ?

The powers of a and b: ?

Pascal's Triangle

In Pascal's Triangle, each term is the sum of the two terms above.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 1 1

[The mathematical secrets of Pascal's Triangle]

The Binomial expansion

Video

This video covers:

- using Pascal's triangle to multiply out expressions of the form $(a+b)^n$, where n is a positive integer

Example

(i) Expand $(p+q)^5$

(ii) Expand $(x+2)^6$

(iii) Expand $(2x-3y)^4$

Your turn…

Write out the following binomial expansions.

(i) $(x+1)^6$

(ii) $(x-2)^5$

(iii) $(2x+1)^4$

(iv) $(3x-2y)^3$

Answers

1. Pascal's triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

(i) $(x+1)^6 = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

(ii)
$$\begin{aligned}(x-2)^5 &= x^5 + 5x^4(-2)^1 + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + (-2)^5 \\ &= x^5 + 5x^4 \times -2 + 10x^3 \times 4 + 10x^2 \times -8 + 5x \times 16 - 32 \\ &= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32\end{aligned}$$

(iii)
$$\begin{aligned}(2x+1)^4 &= (2x)^4 + 4(2x)^3 + 6(2x)^2 + 4(2x) + 1 \\ &= 16x^4 + 4 \times 8x^3 + 6 \times 4x^2 + 8x + 1 \\ &= 16x^4 + 32x^3 + 24x^2 + 8x + 1\end{aligned}$$

(iv)
$$\begin{aligned}(3x-2y)^3 &= (3x)^3 + 3(3x)^2(-2y) + 3(3x)(-2y)^2 + (-2y)^3 \\ &= 27x^3 + 3 \times 9x^2 \times -2y + 3 \times 3x \times 4y^2 - 8y^3 \\ &= 27x^3 - 54x^2y + 36xy^2 - 8y^3\end{aligned}$$

Combinations

Video

This video covers:

- calculating combinations
- using the formula for binomial coefficients

Combinations

$^nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
said "n choose r", is the number of ways of 'choosing' r things from n, such that the order in our selection does not matter.
These are also known as **binomial coefficients**.

For example, if you choose a football team captain and need to choose 4 people from amongst 10 in your class, there are $\binom{10}{4} = \frac{10!}{4!6!} = 210$ possible selections.

(Note: the $\binom{10}{4}$ notation is preferable to $^{10}C_4$)

Use the nCr button on your calculator (your calculator input should display "10C4")

Your Turn

Without using a calculator, evaluate the following binomial coefficients.

(i) ${}_{8}C_{3}$

(ii) ${}_{9}C_{5}$

(iii) ${}_{12}C_{4}$

(iv) ${}_{20}C_{18}$

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Answers

(i) $${}_{8}C_{3} = \frac{8\times 7\times \cancel{6}}{1\times \cancel{2}\times \cancel{3}} = 8\times 7 = 56$$

(ii) $${}_{9}C_{5} = {}_{9}C_{4} = \frac{9\times \cancel{8}^{2}\times 7\times \cancel{6}}{1\times \cancel{2}\times \cancel{3}\times \cancel{4}} = 9\times 2\times 7 = 126$$

(iii) $${}_{12}C_{4} = \frac{\cancel{12}\times 11\times \cancel{10}^{5}\times 9}{1\times \cancel{2}\times \cancel{3}\times \cancel{4}} = 11\times 9\times 5 = 495$$

(iv) $${}_{20}C_{18} = {}_{20}C_{2} = \frac{\cancel{20}^{10}\times 19}{1\times \cancel{2}} = 10\times 19 = 190$$

Using Binomial Coefficients to Expand

$\mathbb{N}$ is the set of natural numbers, i.e. positive integers. This formula is only valid for positive integers n. In Year 2 you will see how to deal with fractional/negative n.

✎ The binomial expansion, when $n \in \mathbb{N}$:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n$$

Find the first 4 terms in the expansion of $(3x+1)^{10}$, in ascending powers of x.

$$(3x+1)^{10} = \binom{10}{0}(1^{10}) + \binom{10}{1}(1^9)(3x)^1 + \binom{10}{2}(1^8)(3x)^2 + \binom{10}{3}(1^7)(3x)^3 + \cdots$$

$$= 1 + 30x + 405x^2 + 3240x^3 + \cdots$$

This is exactly the same method as before, except we've just had to calculate the Binomial coefficients ourselves rather than read them off Pascal's Triangle.

Getting a single term in the expansion

In the expansion of $(a+b)^n$ the general term is given by $\binom{n}{r} a^{n-r} b^r$

Expression	Power of x in term wanted.	Term in expansion
$(a+x)^{10}$	3	?
$(2x-1)^{75}$	50	?
$(3-x)^{12}$	7	?
$(3x+4)^{16}$	3	?

Your turn...

Find

(i) the coefficient of x^3 in the expansion of $(1+3x)^{10}$

(ii) the coefficient of x^5 in the expansion of $(1-2x)^9$

(iii) the coefficient of x^{11} in the expansion of $(2+x)^{15}$

Answers

(i) Term in $x^3 = {}_{10}C_3(3x)^3 = 120 \times 27x^3 = 3240x^3$

Coefficient of x^3 is 3240.

(ii) Term in $x^5 = {}_9C_4(-2x)^5 = 126 \times -32x^5 = -4032x^5$

Coefficient of x^5 is -4032.

(iii) Term in $x^{10} = {}_{15}C_{11} \times 2^4 x^{11} = 1365 \times 16x^{11} = 21840x^{11}$

Coefficient of x^{11} is 21840.

And finally…

The coefficient of x^4 in the expansion of $(1 + qx)^{10}$ is 3360.
Find the possible value(s) of the constant q.

Term is:

?

Therefore:

?

Your turn…

In the expansion of $(1 + ax)^{10}$, where a is a non-zero constant the coefficient of x^3 is double the coefficient of x^2. Find the value of a.

?

Any questions?

BINOMIAL DISTRIBUTION

Lesson 2 How is probability used in real life? (Video)

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DO NOW

A spinner is spun 5 times. Complete this table of probabilities for the random variable X, the number of times **red** appears.

x	0	1	2	3	4	5
P(X = x)	0.07776	0.2592	0.3456	0.2304	0.0768	0.01024

$$P(X = 0) = P(\bar{R}\bar{R}\bar{R}\bar{R}\bar{R}) = \left(\frac{3}{5}\right)^5$$

$$P(X = 1) = P(R\bar{R}\bar{R}\bar{R}\bar{R}) \text{ or } P(\bar{R}R\bar{R}\bar{R}\bar{R}) \text{ or } P(\bar{R}\bar{R}R\bar{R}\bar{R}) \text{ or } P(\bar{R}\bar{R}\bar{R}R\bar{R}) \text{ or } P(\bar{R}\bar{R}\bar{R}\bar{R}R) = 5\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^4$$

$$P(X = 2) = P(RR\bar{R}\bar{R}\bar{R}) \text{ or } P(R\bar{R}R\bar{R}\bar{R}) \text{ or } P(R\bar{R}\bar{R}R\bar{R}) \text{ or } P(R\bar{R}\bar{R}\bar{R}R) \text{ or } P(\bar{R}RR\bar{R}\bar{R}) \text{ or } \ldots = 10\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^3$$

$$P(X = 3) = 10\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^2$$

$$P(X = 4) = 5\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)$$

$$P(X = 5) = \left(\frac{2}{5}\right)^5$$

The Binomial Distribution

- **1, 5, 10, 5, 1** are the numbers in the 6^{th} row of **Pascal's triangle**.

- The number of ways of choosing r reds from 5 spins is: $5Cr$

Links $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

It is sometimes written as nCr or ${}^nC_r$. It represents the number of ways of selecting r successful outcomes from n trials.

On your calculator check 6C2 = 15:

Casio fx-85GT/CLASSWIZZ:
6 SHIFT ÷ 2 =

Casio fx-CG50:
In menu 1, OPTN F6(more options) F3(Probability) F3(nCr)
Use arrow keys to input 6 in front and 2 after then EXE

The Binomial Distribution

Our random variable, X, the number of reds in 5 spins, can be modelled as a binomial distribution. $\mathrm{X} \sim B(n, p)$ if:

- there are a fixed number of trials, n,
- there are two possible outcomes: 'success' and 'failure',
- there is a fixed probability of success, p
- the trials are independent of each other

If $X \sim B(n, p)$ then:

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$$

The Binomial Distribution

Video

Further Examples

The random variable $X \sim B\left(12, \frac{1}{6}\right)$. Find:

a) $P(X = 2)$
b) $P(X = 9)$
c) $P(X \leq 1)$

a	?
b	?
c	?

Your turn…

$X \sim B(8, 0.6)$. Find the following probabilities:

(i) $P(X = 0)$

(ii) $P(X = 3)$

(iii) $P(X = 6)$

Answers

$X \sim B(8, 0.6)$

(i) $P(X = 0) = (0.4)^8 = 0.000655 \ (3 \text{ s.f.})$

(ii) $P(X = 3) = {}_8C_3(0.6)^3(0.4)^5 = \dfrac{8 \times 7 \times 6}{1 \times 2 \times 3}(0.6)^3(0.4)^5 = 0.124 \ (3 \text{ s.f.})$

(iii) $P(X = 6) = {}_8C_6(0.6)^6(0.4)^2 = \dfrac{8 \times 7}{1 \times 2}(0.6)^6(0.4)^2 = 0.209 \ (3 \text{ s.f.})$

Cumulative Probabilities

Often we wish to find the probability of a range of values.
For a Binomial distribution, this was relatively easy if the range was narrow, e.g. $P(X \leq 1) = P(X = 0) + P(X = 1)$, but would be much more computationally expensive if we wanted say $P(X \leq 6)$.

If $X \sim B(10, 0.3)$, find $P(X \leq 6)$.

How to calculate on your fx-CG50:

Press Menu then 'Statistics'
Choose Distribution, Binomial, Bcd
Choose 'Variable'.

$$Lower = 0$$
$$Upper = 6$$
$$N = 10$$
$$p = 0.3$$

Pressing 'EXE' gives the desired value.

Using tables

Look up $n = 10$ and the column $p = 0.3$.
Then look up the row $x = 6$.
The value should be 0.9894.

Important Note: The tables only have limited values of p. You may have to use your calculator.
You will need to use your calculator in the exam anyway.

Tables

Binomial cumulative distribution function

The tabulated value is $P(X \leq x)$, where X has a binomial distribution with index n and parameter p.

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 5, x = 0$	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
$n = 6, x = 0$	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
$n = 7, x = 0$	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
$n = 8, x = 0$	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
$n = 9, x = 0$	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
$n = 10, x = 0$	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 12, x = 0$	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
$n = 15, x = 0$	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 20, x = 0$	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Probabilities

The random variable $X \sim B(20, 0.4)$. Find:

$P(X \leq 7) =$?
$P(X < 6) =$?
$P(X \geq 15) =$?

$n = 20, x = 0$	0.0000
1	0.0005
2	0.0036
3	0.0160
4	0.0510
5	0.1256
6	0.2500
7	0.4159
8	0.5956
9	0.7553
10	0.8725
11	0.9435
12	0.9790
13	0.9935
14	0.9984
15	0.9997
16	1.0000
17	1.0000
18	1.0000

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Given that $X \sim B(25, 0.25)$

$P(X = 6) =$?

$P(X > 20) =$?

$P(6 < X \leq 10)$?

$p =$	0.25
$n = 25,\ x = 0$	0.0008
1	0.0070
2	0.0321
3	0.0962
4	0.2137
5	0.3783
6	0.5611
7	0.7265
8	0.8506
9	0.9287
10	0.9703
11	0.9893
12	0.9966
13	0.9991
14	0.9998
15	1.0000
16	1.0000
17	1.0000
18	1.0000
19	1.0000
20	1.0000
21	1.0000
22	1.0000

Cumulative Probabilities

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Quickfire Questions

Write the following in terms of cumulative probabilities, e.g. $P(X < 7) = P(X \leq 6)$

$P(X < 5) =$?

$P(X \geq 7) =$?

$P(X > 7) =$?

$P(10 \leq X < 20) =$?

$P(10 \leq X \leq 20) =$?

$P(X = 100) =$?

$P(20 < X < 30)$?

“at least 30” = ?

“greater than 30” = ?

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Your turn…

- *Worksheet*

1. A multiple choice test consists of ten questions each of which has five possible answers. A particular student decides to select her answers at random. Find the probability of obtaining
   (i) only one correct answer
   (ii) exactly five correct answers
   (iii) fewer than three correct answers
   (iv) at least two correct answers.

2. In an experiment, a biased coin is thrown ten times. If the probability of obtaining a head on any throw is 0.4, find the probability of obtaining
   (i) fewer than five heads
   (ii) exactly five heads
   (iii) more than three heads.
   (iv) If the above experiment is performed seven times, what is the probability that exactly five heads are obtained on exactly two occasions?

3. A box contains a large number of bulbs. 20% of the bulbs are white, the rest are red. Bulbs are selected at random.
   How many bulbs must be selected so that the probability that there is at least one white bulb is greater than 0.95?

Your turn…

- *Worksheet*

4. Using recent data provided by the low-cost airline Sleezyjet, the probability that a passenger loses his suitcase on a flight is estimated to be 0.15.
   Each week I make six different flights with Sleezyjet.
   (i) In a particular week, find the probability that
      (a) I arrive with my suitcase on all flights,
      (b) I lose my suitcase exactly once,
      (c) I lose my suitcase more than once,
      (d) I lose my suitcase exactly three times.
   (ii) If I fly for four weeks with six flights each week, what is the probability that I arrive with my suitcase on all flights for three weeks out of the four?

5. Past records show that a particular basketball player scores on 35% of her free shots. In practice the player takes sets of 10 free shots.
   (i) Comment on the suitability of using the binomial distribution for modelling the number of times she will score from a set of 10 free shots.
   (ii) Assuming that the binomial distribution is appropriate, find the probability that she will score fewer than four times in a set of 10 free shots.
   (iii) One day the player takes five sets of 10 free shots. Find the probability that she scores fewer than four times in exactly three of the sets.

Answers

1. Let X be the number of correct answers, so X ~ B(10, 0.2)
   (i) $P(X=1) = {}_{10}C_1 \times 0.2^1 \times 0.8^9 = 0.2684$ (4 s.f.)
   (ii) $P(X=5) = {}_{10}C_5 \times 0.2^5 \times 0.8^5 = 0.02642$ (4 s.f.)
   (iii) $P(X<3) = P(X \le 2) = 0.6778$ (4 s.f.)
   (iv) $P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.3758 = 0.6242$ (4 s.f.)

2. Let X be the number of heads obtained, so X ~ B(10, 0.4)
   (i) $P(X<5) = P(X \le 4) = 0.6331$ (4 s.f.)
   (ii) $P(X=5) = {}_{10}C_5 \times 0.4^5 \times 0.6^5 = 0.2007$ (4 s.f.)
   (iii) $P(X>3) = 1 - P(X \le 3) = 1 - 0.3823 = 0.6177$ (4 s.f.)
   (iv) Let Y be the number of occasions on which exactly 5 heads are obtained,
   so Y ~ B(7, 0.2007)
   $P(Y=2) = {}_7C_2 \times 0.2007^2 \times 0.7993^5 = 0.276$ (3 s.f.)

3. Let X be the number of white bulbs.
   X ~ B(n, 0.2)
   $P(X \ge 1) > 0.95$
   $1 - P(X = 0) > 0.95$
   $P(X = 0) < 0.05$
   $0.8^n < 0.05$
   $0.8^{13} = 0.055$ and $0.8^{14} = 0.044$
   The least number of bulbs that must be selected is 14.

4. (i) $X \sim B(6, 0.15)$

(a) $P(X = 0) = 0.85^6 = 0.377$ (3 s.f.)

(b) $P(X = 1) = 6 \times 0.15 \times (0.85)^5 = 0.399$ (3 s.f.)

(c) $P(X > 1) = 1 - P(X \leq 1)$
$= 1 - 0.776$
$= 0.224$ (3 s.f.)

(d) $P(X = 3) = {}_6C_3(0.15)^3(0.85)^3$
$= \frac{6 \times 5 \times 4}{1 \times 2 \times 3}(0.15)^3(0.85)^3 = 0.0415$ (3 s.f.)

(ii) Let Y be the number of weeks in which I arrive with my suitcase on all flights.
$Y \sim B(4, 0.85^6)$
$P(Y = 3) = 4 \times (0.85^6)^3 \times (1 - 0.85^6) = 0.134$

5. (i) Could be argued either way – either that it is reasonable to assume that the each trial
is independent and the probability of success is constant, or that it is not reasonable
as the outcome of each trial could affect the next (improving with practice, or loss of
confidence).

(ii) Let X be the number of scores in 10 free shots, so $X \sim B(10, 0.35)$
$P(X < 4) = P(X \leq 3) = 0.5138$ (4 s.f.)

(iii) Let Y be the number of times that she scores fewer than 4 times in a set, so $Y \sim B(5, 0.5138)$
$P(Y = 3) = {}_5C_3 \times 0.5138^3 \times 0.4862^2 = 0.321$ (3 s.f.)

Any questions?